A Comparative Evaluation of K_{op} Determination and ΔK_{eff} Estimation Methods

Jae-Youn Kang*
Korea Institute of Machinery & Materials, 171 Jang-dong, Yusong, Taejon 305-343, Korea
Ji-Ho Song, Ja-Suk Koo
Department of Automation and Design Engineering, Korea Advanced Institute of Science and Technology, Seoul 130-012, Korea
Byung-Ik Choi
Korea Institute of Machinery & Materials, 171 Jang-dong, Yusong, Taejon 305-343, Korea

Methods for determination of the crack opening stress intensity factor (K_{op}) and for estimation of the effective stress intensity factor range (ΔK_{eff}) are evaluated for crack growth test data of aluminum alloys. Three methods of determining K_{op}, visual measurement, ASTM offset compliance method, and the neural network method proposed by Kang and Song, and three methods of estimating ΔK_{eff}, conventional, the 2/PI0 and 2/PI methods proposed by Donald and Paris, are compared in a quantitative manner by using evaluation criteria. For all K_{op} determination methods discussed, the 2/PI method of estimating ΔK_{eff} provides good results. The neural network method of determining K_{op} provides good correlation of crack growth data. It is recommended to use 2/PI estimation with the neural K_{op} determination method. The ASTM offset method used in conjunction with 2/PI estimation shows a possibility of successful application. It is desired to improve the ASTM method.

Key Words : Crack Closure, Effective Stress Intensity Factor Range, Evaluation Criteria, Fatigue Crack Growth, Opening Load

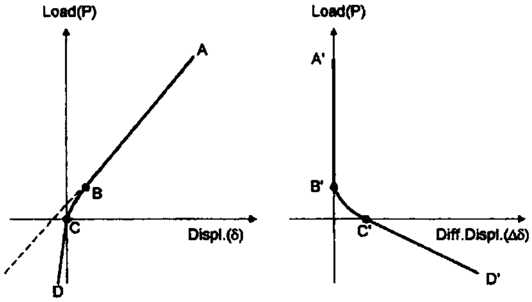
1. Introduction

It has been widely recognized that fatigue crack growth rate can be well expressed in terms of the effective stress intensity factor range ΔK_{eff} based on the crack closure concept. (Bruzzi and McHugh, 2003 ; Choi, 2000 ; Lee and Chen, 1999) When using ΔK_{eff} for estimating fatigue crack growth rate da/dN, the most important and difficult part is to measure crack closure precisely and determine consistently the crack opening stress intensity factor K_{op}.

Although many experimental techniques have been developed to measure crack closure, the so-called compliance technique is most widely used, due to its experimental simplicity and low-cost. There are two methods in the technique : One is the conventional compliance method originally used by Elber (1971) which utilizes the load-displacement curve as shown in Fig. 1(a), and the other is the unloading elastic compliance method first proposed by Kikukawa et al. (1976) which utilizes the load-differential displacement curve as shown in Fig. 1(b) to improve measurement sensitivity and precision.

Usually the crack opening load is determined by visual measurement, but visual measurement

* Corresponding Author,
E-mail : jykang@kimm.re.kr
TEL : +82-42-868-7850; **FAX** : +82-42-848-7884
Korea Institute of Machinery & Materials, 171 Jang-dong, Yusong, Taejon 305-343, Korea. (Manuscript **Received** November 27, 2003; **Revised** March 3, 2004)



(a) (b)

Fig. 1 Typical curves of (a) Load versus displacement and (b) Load versus differential displacement

is likely to depend on the observer's experience and provide inconsistent results. Several methods have been proposed to determine the crack opening load and particularly, ASTM (ASTM E647-99, 2001) has recommended the so-called compliance offset method. This method can be used easily and conveniently, due to its procedural simplicity. However, due to the employment of the offset compliance approach and a conventional compliance method, it has some drawbacks that the crack opening load will be underestimated and the measurement precision is relatively poor.

Kang and Song (1998) have proposed an automated crack opening load determination method using a neural network. By using a neural network, the method can determine precisely the crack opening load from differential displacement signal curve automatically uninfluenced by the observer. A disadvantage is that the method requires training a neural network with ideal differential displacement signal patterns.

As the ASTM offset method and the neural one can determine consistently the crack opening load, they are utilized to determine the crack opening load, along with visual measurement, in this study.

The effective stress intensity factor range ΔK_{eff} based on the crack closure concept is conventionally defined as

$$\Delta K_{eff} = K_{max} - K_{op} \qquad (1)$$

where K_{max} and K_{op} denote the maximum stress intensity factor and the crack opening stress intensity factor, respectively.

It has been frequently pointed out that the conventional ΔK_{eff} estimated by equation (1) tends to give an underestimate of the substantial effective stress intensity factor range and cannot express fatigue crack growth rates, particularly in the near-threshold regime (Chen et al., 1996; Donald, 1997). Donald (1997) has shown that significant crack tip strain can occur below the crack opening load and noted that additional driving force below the conventional opening load may contribute to the crack growth. Based on this argument, he proposed an alternative ΔK_{eff} estimation method called the adjusted compliance ratio (ACR) method. An important advantage of the ACR method is that this method uses only the values of slope and secant of the load-displacement curve for ΔK_{eff} estimation and does not require measurement of the crack opening load.

In the recent, recognizing the argument of Donald described above and noting that a simple physical model has not been discovered for the basis of ACR method, Paris, Tada and Donald (1999) proposed the so-called $2/\pi 0$ and $2/\pi$ correction method based on the partial crack closure model. They have assumed that partial crack closure occurs due to the crack surface roughness at a small distance behind the crack tip and proposed a simple mathematical model to estimate the effect of partial crack closure on the effective stress intensity factor. The $2/\pi 0$ and $2/\pi$ correction methods estimate ΔK_{eff} approximately by applying the $2/\pi$ factor on the conventionally measured K_{op}.

Investigating various methods of estimating ΔK_{eff}, Donald and Paris (1999) have reported that the conventional ΔK_{eff} estimation method appears adequate at crack growth rates above 1×10^{-5} mm/cycle, while the $2/\pi 0$ or $2/\pi$ methods appear to provide a successful correlation of the crack growth rate data in the near-threshold regime at crack growth rates below 10^{-7} mm/cycle.

Recently, Kujawski (2001) has suggested an enhancement of Donald and Paris' partial crack closure model, using a transition function to evolve from Donald and Paris' partial crack closure model at the near-threshold region to the conventional Elber's closure approach at the Paris region.

The results described hitherto imply that the $2/\pi0$ and $2/\pi$ correction methods are useful to estimate ΔK_{eff}. However, a systematic quantitative evaluation of the methods has not been reported yet.

In this work, the ΔK_{eff} estimation methods due to the $2/\pi0$ and $2/\pi$ correction are extensively evaluated along with the conventional ΔK_{eff} estimation, in relation to various K_{op} determination methods described in the above.

Particularly, in order to evaluate K_{op} determination and ΔK_{eff} estimation methods on a quantitative basis, some evaluation criteria are introduced. The validity of methods is evaluated.

2. K_{op} Determination Methods

In this study, the ASTM compliance offset method and the neural network method are used to determine K_{op}, along with visual measurement. Since each method is described in detail in the related literatures (ASTM E647-99, 2001 ; Kang and Song, 1998) only the important aspects are outlined here.

2.1 The ASTM compliance offset method

The load-displacement data are collected for a complete load cycle. On the unloading curve, a least squares straight line is fitted to the upper segment of the curve that spans a range of approximately 25% of the cyclic load range. The slope of this line is assumed to be the compliance value that corresponds to the fully-open crack configuration. Next, on the loading part of a load-displacement curve, least-squares straight lines are fitted to segments of curve that span a range of approximately 10% of the cycle load range and that overlap each other by approximately 5% of the cyclic load range, as shown in Fig. 2(a). The compliance offset is calculated by comparing the compliance of each segment with the open-crack compliance. As shown in Fig. 2

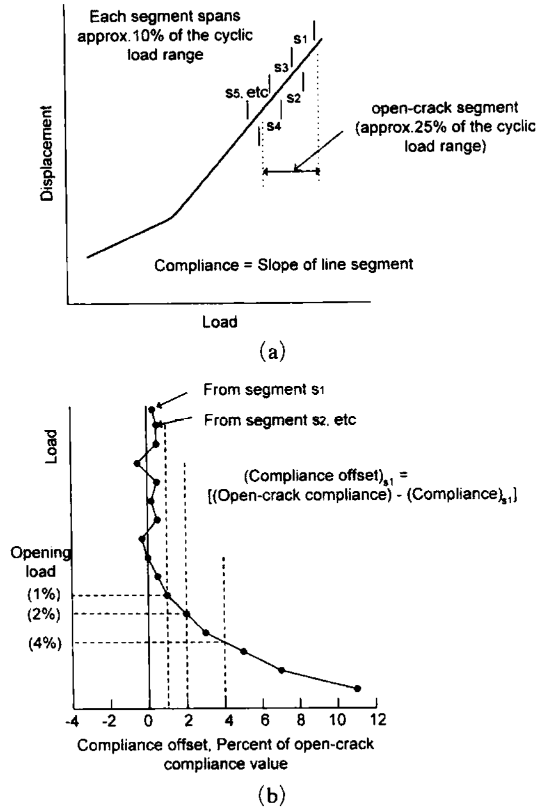


Fig. 2 Determination of opening load due to the ASTM compliance offset method

(b), the opening load is determined by 1, 2, or 4% compliance offset criteria.

As the 2% offset criterion is typically used and is considered to provide practically good results, the 2% offset criterion is employed to determine the opening load in this study.

2.2 The neural network method

As briefly described in the introduction, load-differential displacement curve has been frequently used to determine crack opening point precisely. When the baseline load-differential displacement curve as shown in Fig. 1(b) is obtained, the differential displacement signal $D_1'B_1'A'B_2'D_2'$ may be depicted schematically as shown in Fig. 3. If there is assumed to be no phase difference in the differential displacement between the loading and unloading stages for convenience, the linear (crack fully open) portion

$B_1'A'B_2'$ of the differential displacement signal will be straight. Accordingly the crack opening load P_{op} can be determined theoretically as the load corresponding to the point B_1' deviating from the linear portion $B_1'A'B_2'$. In order to determine automatically the crack opening point B_1' on the differential displacement signal curve, Kang and Song employed a back propagation neural network as shown in Fig. 4. 100 data points of the differential displacement signal on the loading stage (the portion $D_1'B_1'A'$) are used for the input data of the neural network. Accordingly, the number of input units is n=100. Only one hidden layer is used and the number of hidden units is p=5. The output of the neural network is the crack opening point corresponding to the point B_1'. Therefore, the number of output units is m=1.

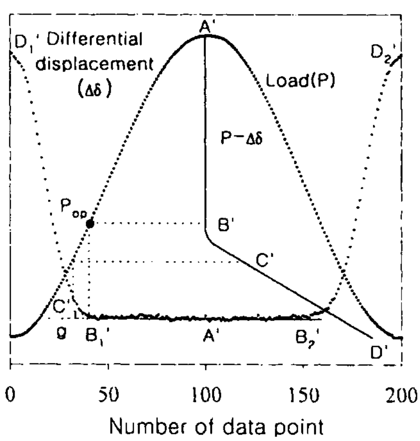


Fig. 3 Scheme of crack opening load determination on differential displacement data

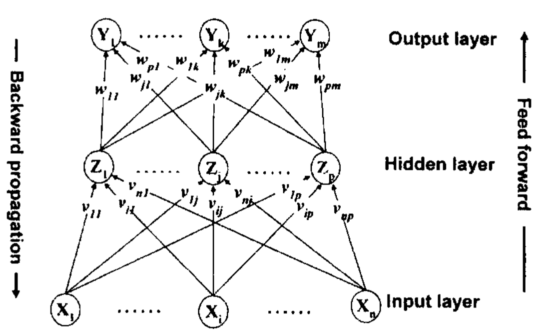


Fig. 4 Back-propagation neural network with one hidden layer

In using neural networks for automatic measurements of crack opening load, it is necessary to train and test a neural network with ideal differential displacement signal patterns in which the true crack opening point can be clearly identified. As it is almost impossible to obtain such ideal differential displacement signal patterns from experimental measurements, Kang and Song simulated signal patterns for training and testing of a neural network by computer using the method proposed by Kim and Song (1993).

Figure 5(a) shows an ideal differential displacement signal represented approximately by combining a sinusoidal wave and a horizontal line. The dashed line in the Figure represents the load signal. The load-differential displacement curve for this example is shown in Fig. 5(b), where the short horizontal bar represents the crack opening point.

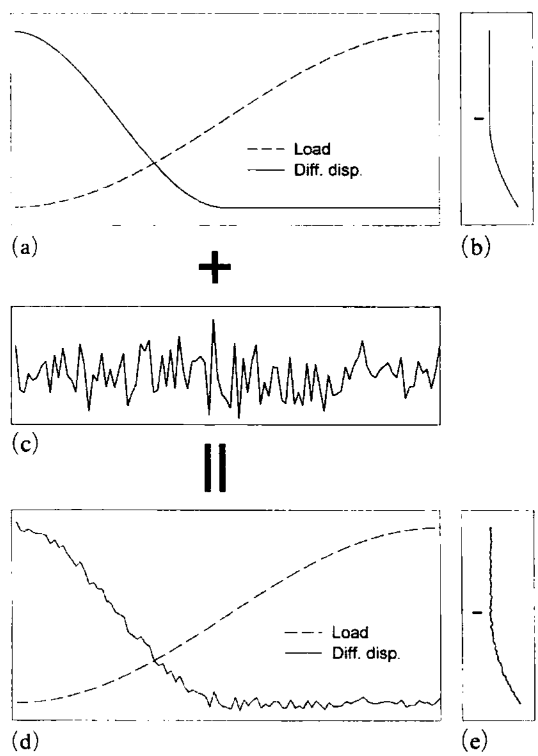


Fig. 5 Generation of simulated differential displacement signal for training a neural network

Since measured signals normally include noise, random noise to be superimposed on the approximated differential displacement signal for training and testing of a neural network, was generated by using a series of sinusoidal waves.

Generated random noise is shown in Fig. 5(c). Superimposing the random noise on the approximated differential displacement signal shown in Fig. 5(a) results in Fig. 5(d). The corresponding load-differential displacement curve is shown in Fig. 5(e).

A neural network was trained and tested, using several hundred different differential displacement signal patterns of various combinations of crack opening levels (40 levels) and signal-to-noise (S/N) ratios (100, 30, 20, 15, 10 dB). The precision of crack opening point measurements by the neural network is found to be dependent on the S/N ratio. For precise measurements within a small error of 3%, the S/N ratio of differential displacement signal is recommended to be higher than 15 dB As the neural network method does not depend on the observer, the method is expected to give consistent and unbiased results.

3. ΔK_{eff} Estimation Methods

In this study, three estimation methods are used to estimate ΔK_{eff}; conventional one expressed by equation (1), the 2/PI0 and 2/PI methods proposed by Donald and Paris.

The latter two methods have been described in detail in the literature (Paris et al., 1999). Therefore, only the most important aspects are described below.

3.1 The 2/PI0 method

Paris, Tada and Donald (1999) have pointed out that if partial crack closure occurs particularly in the near-threshold regime, the minimum effective K for the load cycle may be approximately reduced by the factor $2/\pi$ from the opening K and have suggested that the effective stress intensity factor range may be estimated approximately as

$$\Delta K_{eff}=K_{max}-\frac{2}{\pi}K_{op}\equiv(\Delta K_{eff})_{2/PI0} \qquad (2)$$

To bound the value of the estimated ΔK_{eff} by the applied ΔK, if $\Delta K_{eff}>\Delta K(=K_{max}-K_{min})$, then $\Delta K_{eff}=\Delta K$.

The ΔK_{eff} estimation based on equation (2) is called the $2/\pi 0$ correction and is frequently expressed as $\Delta K_{2/\pi 0}$ or $\Delta K_{2/PI0}$. In this study, the ΔK_{eff} estimation will be hereafter expressed as $(\Delta K_{eff})_{2/PI0}$, as shown in equation (2), to distinguish from other ΔK_{eff} estimations, and the estimation method is referred to as the 2/PI0 method.

3.2 The 2/PI method

As the estimation by equation (2) gives an overestimate of ΔK_{eff}, Paris, Tada, and Donald (1999) have proposed another one called as $2/\pi$ correction expressed as

$$\Delta K_{eff}=K_{max}-\frac{2}{\pi}K_{op}-\left(1-\frac{2}{\pi}\right)K_{min} \equiv(\Delta K_{eff})_{2/PI} \qquad (3)$$

where K_{min} is the minimum stress intensity factor. For negative stress ratios, K_{min} is assumed to be zero.

This method has a merit that the estimated ΔK_{eff} is bounded by ΔK without any particular assumption. The estimated ΔK_{eff} by equation (3) is hereafter expressed as $(\Delta K_{eff})_{2/PI}$ and the corresponding estimation method is referred to as the 2/PI method.

4. Evaluation Criteria

Figure 6(a) shows the crack growth data of 2024-T351 aluminum alloy at stress ratios of $R=-1$, -0.5, 0, 0.1 and 0.3 obtained by one of the authors (Kim and Song, 1993; Lee and Song, 2000). The crack growth rate da/dN as a function of ΔK is strongly influenced by the stress ratio. Figure 6(b), (c) and (d) show the crack growth rates as a function of conventional ΔK_{eff} estimated by equation (1) based on K_{op} values determined by visual measurement by well-trained observers, the ASTM offset method and the neural method, respectively. It appears that the stress ratio effects are well correlated by the conventional ΔK_{eff} based on K_{op} values by visual measurement or by the neural method, while the results by the ASTM method are slightly widely dispersed. Consequently, it may be said in qualitative manner that the ASTM method is inferior to the other two methods, but it is not clear whether is better, visual measurement or the neural method.

In this study, in order to evaluate K_{op} determination and ΔK_{eff} estimation methods on a quantitative basis, some evaluation criteria are introduced. One is the following error criterion: If the crack growth rates are represented in terms of ΔK_{eff} obtained by a certain combination of K_{op} determination and ΔK_{eff} estimation methods, as illustrated in Fig. 7, the error criterion is expressed in terms of the fraction of data falling within a scatter band of a specified factor s around the regression lines as

$$E_f(s=\sqrt{2}) = \frac{\text{Number of data falling within } \frac{1}{\sqrt{2}} \leq \frac{\left(\frac{da}{dN}\right)_{observed}}{\left(\frac{da}{dN}\right)_{regression\ line}} \leq \sqrt{2}}{\text{Number of total data}} \quad (4)$$

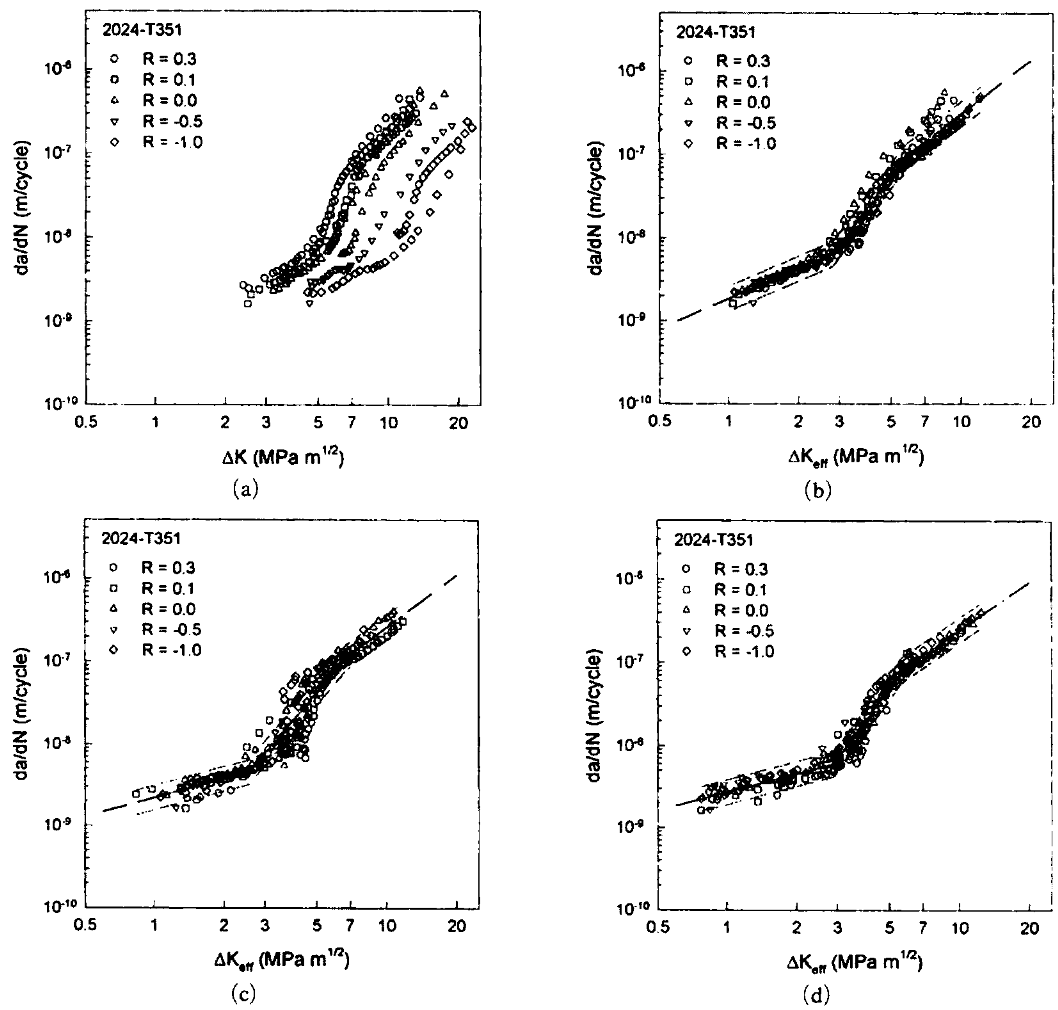


Fig. 6 Fatigue crack growth rates on 2024-T351 (a) As a function of ΔK and as a function of ΔK_{eff} based on (b) Visual K_{op}, (c) ASTM 2% K_{op}, (d) Neural K_{op}

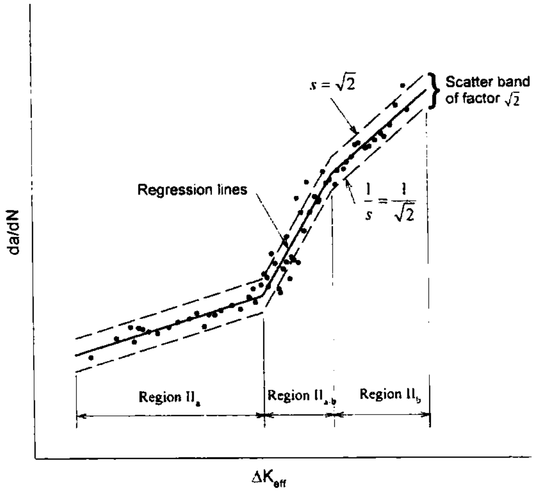


Fig. 7 Illustration of error criterion used

where $\left(\frac{da}{dN}\right)_{observed}$ and $\left(\frac{da}{dN}\right)_{regression\ line}$ denote the observed crack growth rate and the estimated one from the regression line, respectively. The regression lines are obtained separately for three growth rate regimes as shown in Fig. 7. Here, a value of s of $\sqrt{2}$ is employed.

As the error criterion cannot evaluate the correlation of da/dN versus the estimated ΔK_{eff}, the correlation coefficient r^2 is utilized as an additional evaluation criterion. As the correlation coefficient r^2 varies depending on the growth rate regime, the representative value of correlation coefficient r^2_{total} is calculated as

$$r^2_{totol}=\frac{\sum_{i=1}^{k}N_i r_i^2}{\sum_{i=1}^{k}N_i} \tag{5}$$

where r_i^2 denotes the correlation coefficient for the i-th growth rate regime and N_i, the number of data included in the i-th growth rate regime and k, the number of growth rate regimes.

K_{op} determination and ΔK_{eff} estimation methods are evaluated finally by the following evaluation value $\bar{E}$ averaged over the above two evaluation items as

$$\bar{E}=\frac{E_f(s=\sqrt{2})+r^2_{total}}{2} \tag{6}$$

The closer the evaluation value is to 1, the better the method used is.

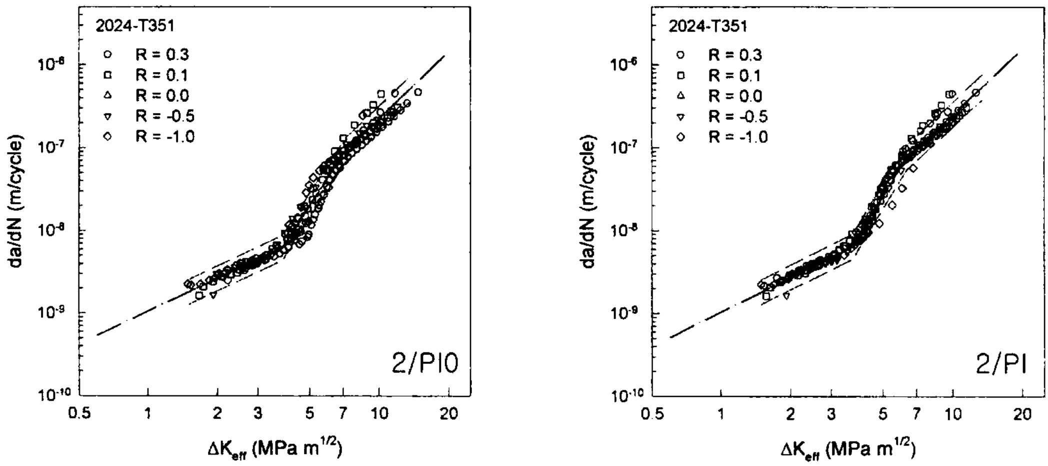
5. Evaluation of K_{op} Determination and ΔK_{eff} Estimation Methods

The crack growth data shown in Fig. 6 are reexamined using the 2/PI0 and 2/PI estimation methods and the results are shown in Fig. 8. When K_{op} is determined by visual measurement or the neural method, there is no significant deference among the results by the 2/PI0, 2/PI estimation methods and those by the conventional ΔK_{eff} estimation shown in Fig. 6. However, close inspection of Fig. 8(c) indicates that the 2/PI method used with the neural method reduces the data scatter in the low growth rate region slightly, compared with other methods.

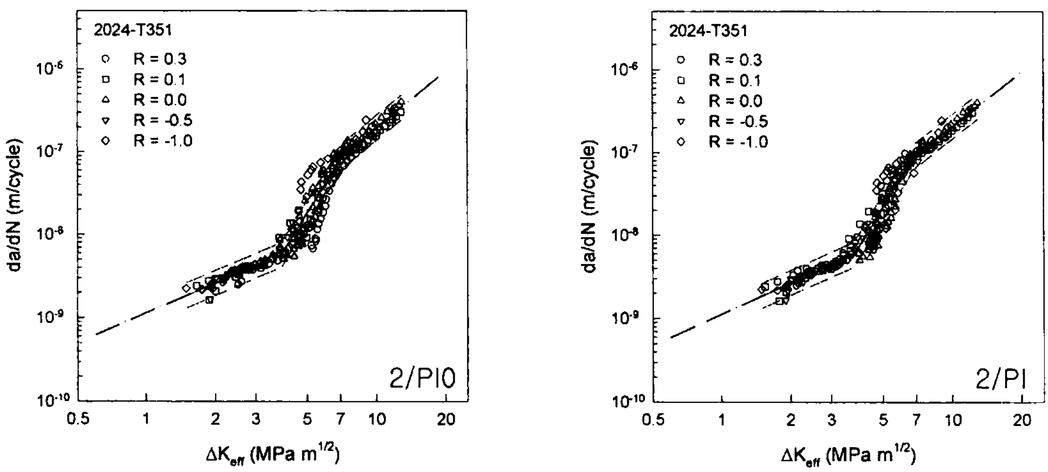
On the other hand, when K_{op} is determined by the ASTM offset method, the 2/PI0 and 2/PI estimation methods reduce the data scatter so significantly to provide a fairly good correlation of the data.

Figure 9 shows the results for the crack growth data of 7075-T6 aluminum alloy of Kim and Song (1992). Excepted for the results based on K_{op} values determined by the ASTM offset method, all results show good correlation of the stress ratio effects. The ASTM offset K_{op} method used with the conventional ΔK_{eff} estimation gives very widely dispersed results, but if the ASTM method is used with the 2/PI ΔK_{eff} estimation, the data scatter is reduced so remarkably to possibly obtain a good correlation of the crack growth rate data.

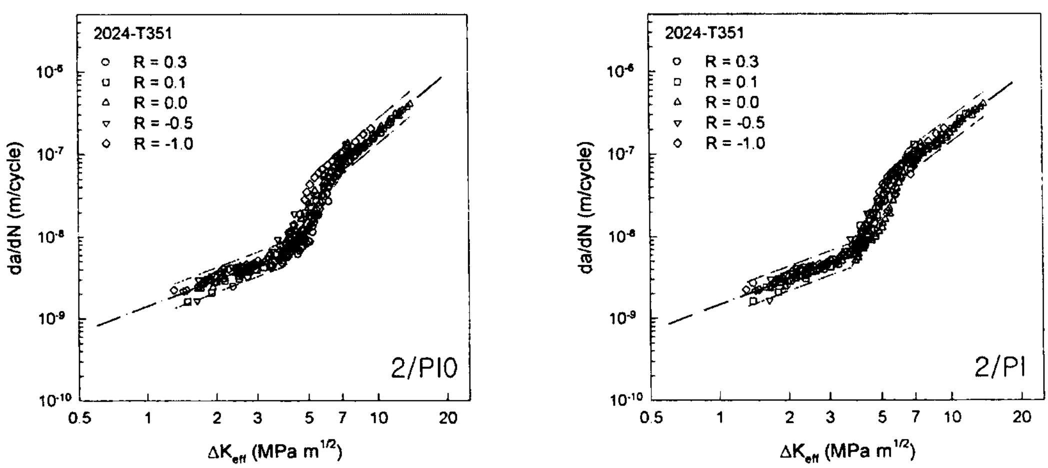
Table 1 shows comparisons of the methods on the basis of the evaluation values defined in the previous section. The neural one among the K_{op} determination methods is found to provide the best results, irrespective of ΔK_{eff} estimation method. Among the ΔK_{eff} estimation methods, the 2/PI method may be considered the best. The 2/PI0 method tends to give lower $\bar{E}$ values, in comparison with the conventional one. When the ASTM K_{op} method is used with the conventional ΔK_{eff} estimation, the value of $\bar{E}$ is relatively low, but when used with the 2/PI estimation, the value of $\bar{E}$ is significantly improved.



(a) Based on visual K_{op}



(b) Based on ASTM 2% K_{op}



(c) Based on neural K_{op}

Fig. 8 Comparison of K_{op} determination and K_{eff} estimation methods for constant amplitude data of 2024-T351 aluminum alloy

6. Discussion

As can be found in Table 1, the neural network method of determining K_{op} provides comparable or better results than visual measurements by well-trained observers, irrespective of ΔK_{eff} estimation method.

On the other hand, the ASTM offset method used with the conventional ΔK_{eff} estimation is almost useless because of the extremely large data scatter as can be seen in Fig. 9. However, if the ASTM method is used, particularly with the 2/PI ΔK_{eff} estimation, the data scatter is significantly reduced and the correlation of stress ratio effects is also considerably improved. If the 2/PI estimation method is used with visual measurement or the neural K_{op} determination method, the correlation of data represented by the value of $\bar{E}$ is improved, but the amount of improvement is

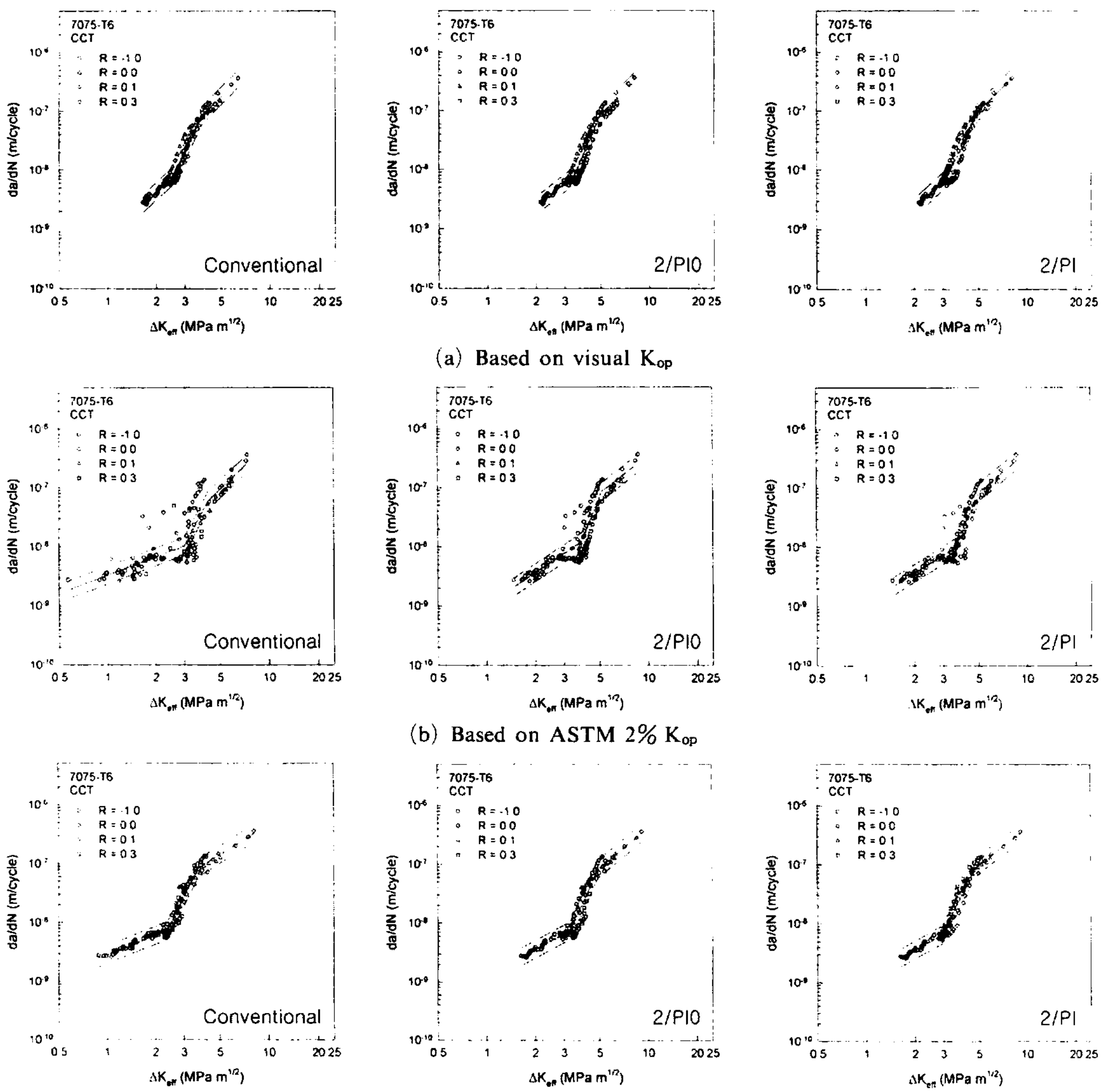


Fig. 9 Comparison of K_{op} determination and K_{eff} estimation methods for constant amplitude data of 7075-T6 aluminum alloy

Table 1 Comparison of K_{op} determination and ΔK_{eff} estimation methods for constant amplitude data

Material		2024-T351			7075-T6		
		ΔK_{eff} evaluation			ΔK_{eff} evaluation		
K_{op} determination		Conventional	2/PI0	2/PI	Conventional	2/PI0	2/PI
Visual	E_f	0.836	0.788	0.840	0.851	0.769	0.752
	r_2	0.792	0.813	0.835	0.852	0.828	0.862
	$\bar{E}$	0.814	0.801	0.838	0.851	0.798	0.807
ASTM 2% offset	E_f	0.671	0.765	0.778	0.512	0.595	0.719
	r_2	0.704	0.827	0.838	0.431	0.561	0.642
	$\bar{E}$	0.687	0.796	0.808	0.472	0.578	0.680
Neural method	E_f	0.868	0.829	0.863	0.892	0.783	0.875
	r_2	0.826	0.840	0.875	0.871	0.846	0.896
	$\bar{E}$	0.847	0.835	0.869	0.881	0.815	0.885

very small. On the other hand, the 2/PI0 estimation method may be said to be inferior, even to conventional ΔK_{eff} estimation.

In conclusion, the neural K_{op} determination method and the 2/PI ΔK_{eff} estimation method are recommended for correlating the stress ratio effects of constant amplitude loading data.

From the evaluation results hitherto described, it can be said that the conventional ΔK_{eff} estimation is not always relevant and the use of the 2/PI method is recommended.

The neural K_{op} determination method is also recommended.

It is worthy of note that the ASTM offset method, if used with the 2/PI estimation method, provides not so bad results. The ASTM offset method has some intrinsic drawbacks that the crack opening load will be underestimated and there is large variability in the results. If these drawbacks can be overcome, the ASTM method may be promising.

7. Conclusions

Three methods of determining K_{op} and three methods of estimating ΔK_{eff} are evaluated in a quantitative manner by using evaluation criteria. The conclusions obtained are summarized as follows:

(1) Irrespective of K_{op} determination method, the 2/PI method of estimating ΔK_{eff} provides good results. It is recommended to use the 2/PI estimation method, instead of the conventional one.

(2) The neural network method of determining K_{op} provides the best stress ratio correlation for all ΔK_{eff} estimation procedures. As the neural method can determine the crack opening load automatically and consistently unaffected by the observer, the use of the method is recommended.

(3) The ASTM offset K_{op} method used in conjunction with 2/PI estimation shows a possibility of successful application. It is desired to improve the method to overcome its intrinsic drawbacks of underestimating K_{op} and resulting in large data scatter.

Acknowledgment

This paper was performed for the Smart UAV Development Program, one of the 21st Century Frontier R&D Programs funded by the Ministry of Science and Technology of Korea.

References

ASTM E647-99, 2001, Standard Test Method for Measurement of Fatigue Crack Growth Rates, In Annual Book of ASTM Standards, Vol. 03.01.

Bruzzi, M. S. and McHugh, P. E., 2003, "Application of Closure Based Fatigue Modelling Methodology to Al-SiC MMCs," *Int J Fatigue*, Vol. 25, pp. 577~584.

Choi, H. C., 2000, "A Study on the Determination of Closing Level for Finite Element Analysis of Fatigue Crack Closure," *KSME International Journal*, Vol. 14, pp. 401~407.

Chen, D. L., Weiss, B. and Stickler, R., 1996, "Contribution of the Cyclic Loading Portion below the Opening Load to Fatigue Crack Growth," *Matererial Science and Engineering A*, Vol. 208, pp. 181~187.

Donald, J. K., 1997, "Introducing the Compliance Ratio Concept for Determining Effective Stress Intensity," *Int J Fatigue*, Vol. 19, pp. s191~s195.

Donald, J. K. and Paris, P. C., 1999, "An Evaluation of ΔK_{eff} Estimation Procedure on 6061-T6 and 2024-T351 Aluminum Alloys," *Int J Fatigue*, Vol. 21, pp. s47~s57.

Elber, W., 1971, "The Significance of Fatigue Crack Closure. In : Damage Tolerance in Aircraft Structures," ASTM STP 486, Philadelphia (PA), American Society for Testing and Materials, pp. 230~242.

Kikukawa, M., Jono, M. and Tanaka, K., 1976, "Fatigue Crack Closure Behavior at Low Stress Intensity Levels," *In Proc. ICM 2*, pp. 716~720.

Kang, J. Y. and Song, J. H, 1998, "Neural Network Applications in Determining the Fatigue Crack Opening Load," *Int J Fatigue*, Vol. 20, pp. 57~69.

Kim, J. H. and Song, J. H.,. 1992, "Crack Growth and Closure Behaviour of Surface Cracks under Axial Loading," *Fat Fract Engng Mater Struct*, Vol. 15, pp. 477~489.

Kim, C. Y. and Song, J. H., 1993, "An Automated Procedure for Determining Crack Opening Level from Differential Displacement Signal Data," *Int J Fatigue*, Vol. 15, pp. 301~309.

Kim, C. Y. and Song, J. H., 1994, "Fatigue Crack Closure and Growth Behavior under Random Loading," *Engng Fract Mech*, Vol. 49, pp. 105~120.

Kujawski, D., 2001, "Enhanced Model of Partial Crack Closure for Correlation of R-ratio Effects in Aluminum Alloys," *Int J Fatigue*, Vol. 23, pp. 95~102.

Lee, S. Y. and Song, J. H., 2000, "Crack Closure and Growth Behavior of Physically Short Fatigue Cracks under Random Loading," *Engng Fract Mech*, Vol. 66, pp. 321~346.

Lee, O. S. and Chen, Z. W., 1999, "A New Crack Growth Model for Life Prediction Under Random Loading," *KSME International Journal*, Vol. 13, pp. 905~911.

Paris, P. C., Tada, H. and Donald, J. K, 1999, "Service Load Fatigue Damage-a Historical Perspective," *Int J Fatigue*, Vol. 21, pp. s35~s46.